LETTER TO THE EDITOR

Quantized resistivity from statistical reaction theory

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Abstract. The Landauer formula for quantized resistivity is derived from basic statistical reaction theory.

The recent observation of quantized conductivity [1], beautifully confirming Landauer's result [2], may give interest to a simplified derivation of the formula based on statistical reaction theory. The central result of the theory is the transition state formula for decay rates. According to the formula, the rate W at which an equilibrated quantum system A decays is

$$W_A = \sum T/(2\pi\hbar \, dn_A/dE). \tag{1}$$

Here the $\sum T$ are the transmission coefficients in the channels, summed over channel states, and dn_A/dE is the density of levels in system A. Channels are defined as the distinct states of the system at a fixed value of the reaction coordinate. This is usually at a barrier top, although the validity of the theory does not depend on the existence of a potential barrier. The transmission coefficients depend only on the energy of the system and take values 1 or 0 in the classical limit for energetically allowed or forbidden decays. Equation (1) expresses the RRKM transition state theory of chemical reactions [3]. It is also well known in nuclear physics, having first been applied in 1937 to neutron evaporation [4].

For the present application, I imagine two capacitor plates joined by a quantum resistor. The rate of flow of electrons from one plate to the other will be calculated, assuming that each plate emits electrons (through the resistor) as a decaying quantum system. The electrons are assumed to move in a common single-particle potential, in which case equation (1) can be applied to the single-particle states to find the current. For electrons leaving plate A this reads

$$I_A = e \sum_n W_A f(\varepsilon_n - \mu_A) \tag{2}$$

where the sum is over electron states of A. In this equation f is the occupation probability of those states, depending on the chemical potential μ. Substituting in equation (1) and

replacing the sum by an integral, it is immediately apparent that the density-of-states factor converts the integration variable to energy,

$$I_A = e \int de \left(\sum T/2\pi\hbar \right) f(\varepsilon - \mu_A). \tag{3}$$

The net current is the difference in currents from the two directions. Since the transmission coefficient does not depend on the direction of flow, the only difference in currents is due to the chemical potentials. This difference may be expressed in terms of the voltage difference V, $\mu_A - \mu_B = eV$. The result is

$$I = I_A - I_B = e \int de \left(\sum T/2\pi\hbar \right) (f(e - \mu_A) - f(\varepsilon - \mu_B)) \sim e^2 \left(\sum T/2\pi\hbar \right) V.$$

The last equality holds when f is a step function on the scale of variations in $T(\varepsilon)$. To get the final result, note that in the absence of magnetic fields the channels will be open in pairs (spin up and spin down). Neglecting quantum barrier transmission effects, the minimum non-vanishing conductivity G occurs when $\sum T = 2$, giving $G = e^2/\pi\hbar$.

References

[1] van Wees B J *et al* 1988 *Phys. Rev. Lett.* **60** 848
    Wharam D *et al* 1988 *J. Phys. C: Solid State Phys.* **21** L209
[2] Landauer R 1970 *Phil. Mag.* **21** 863
[3] See, for example,
    Hase W 1976 *Dynamics of Molecular Collisions* ed W Miller (New York: Plenum) p 125, equation (11)
[4] Weisskopf V 1937 *Phys. Rev.* **52** 295